Problem 12
Resonating Glasses

International Physics Tournament 2020 ▪ University of California, Berkeley
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Problem

When you take two glasses between your fingers, they sometimes emit a particular sound containing a frequency sweep. Investigate the phenomenon.

To investigate phenomena

1. Simplify Model
2. Analyze Data
3. Extend Considerations

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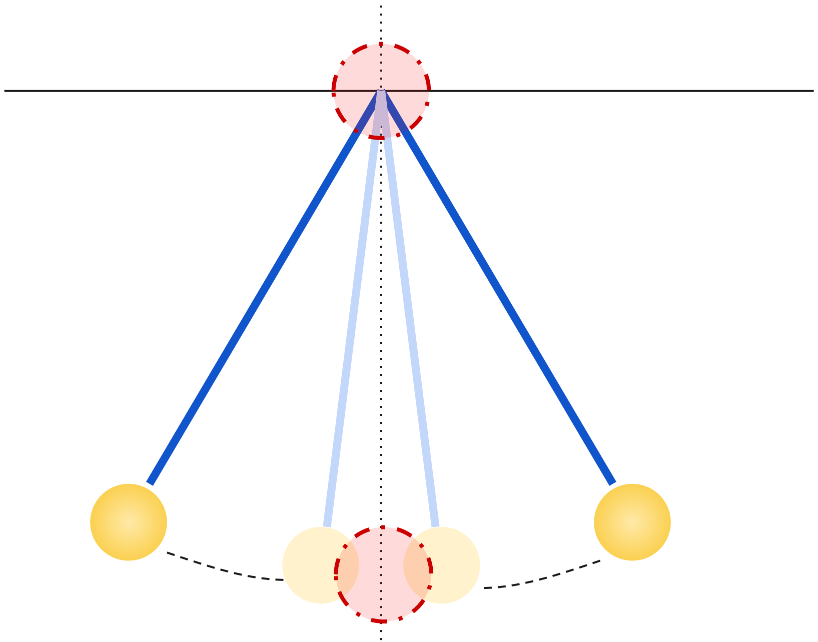
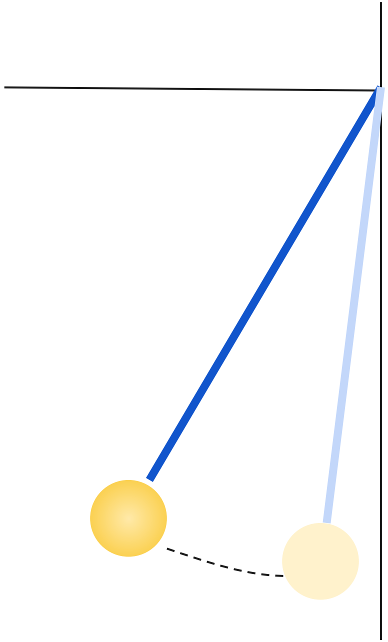
Simplify as a problem of pendulums

Rigid body: For visual purposes, we draw the ‘ball’ at the center of mass.

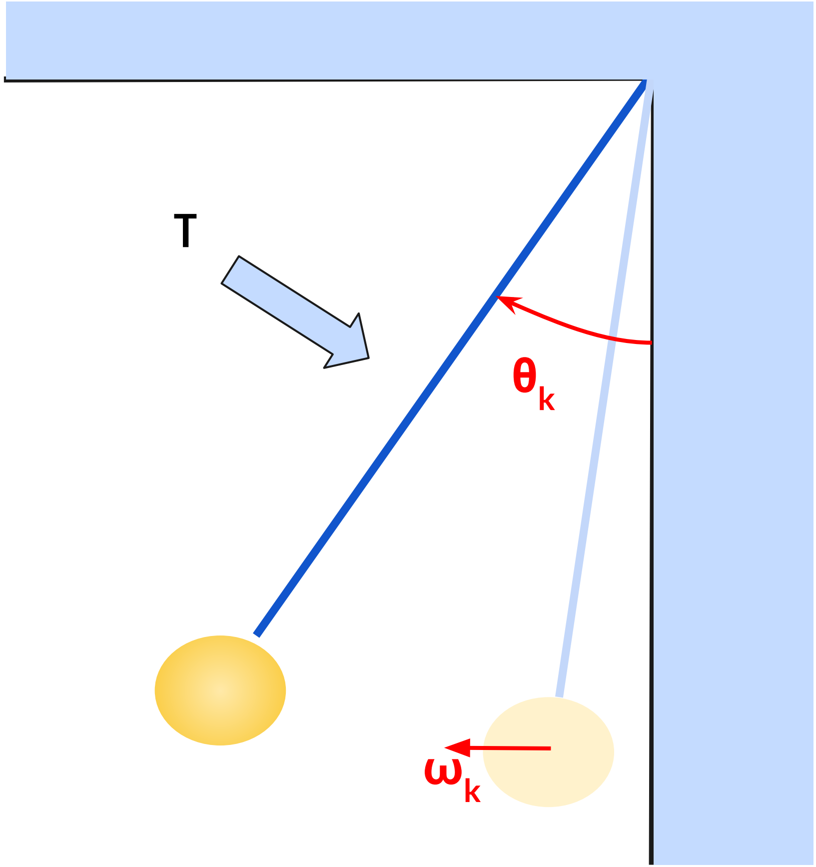


Further reduce to single pendulum & wall



Relevant parameters for successive collisions.

- θ_k ~ max angle before k^{th} collision
- ω_k ~ angular velocity immediately after the $(k-1)^{th}$ collision



What does the system look like for energy?

	Initial Max Angle	Immediately Before Collision	Immediately After Collision	Next Max Angle
Kinetic Energy	0	KE_{k-1}	$KE_k = c^*KE_{k-1}$	0
Potential Energy	$L^*mg(1-\cos(\theta_{k-1}))$	0	0	$L^*mg(1-\cos(\theta_{k-1}))$

Using the small angle approximation along with W=ΔKE

$$\theta_{k+1} = \frac{c \cdot KE_k}{\tau}$$

$$\omega_{k+1} = \sqrt{\frac{2C}{I} \cdot KE_k}$$

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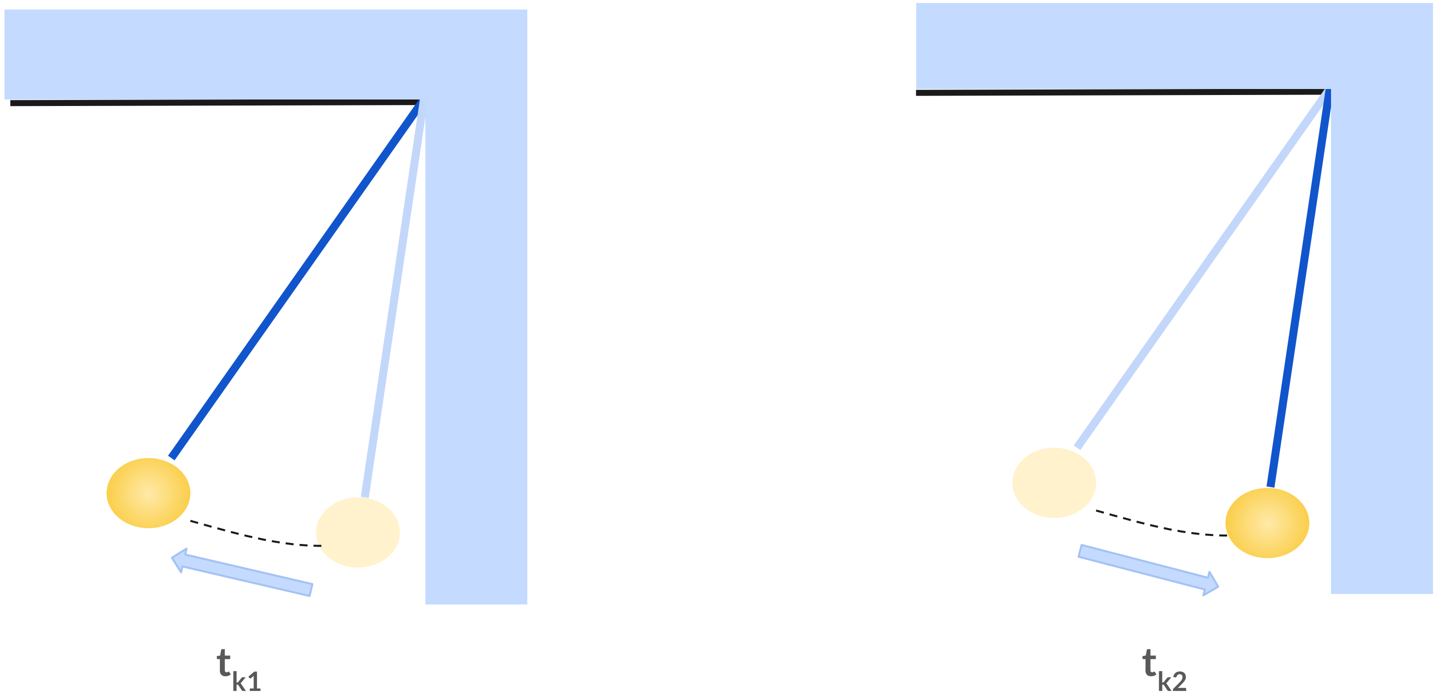
Analysis: Time in two parts



Part I

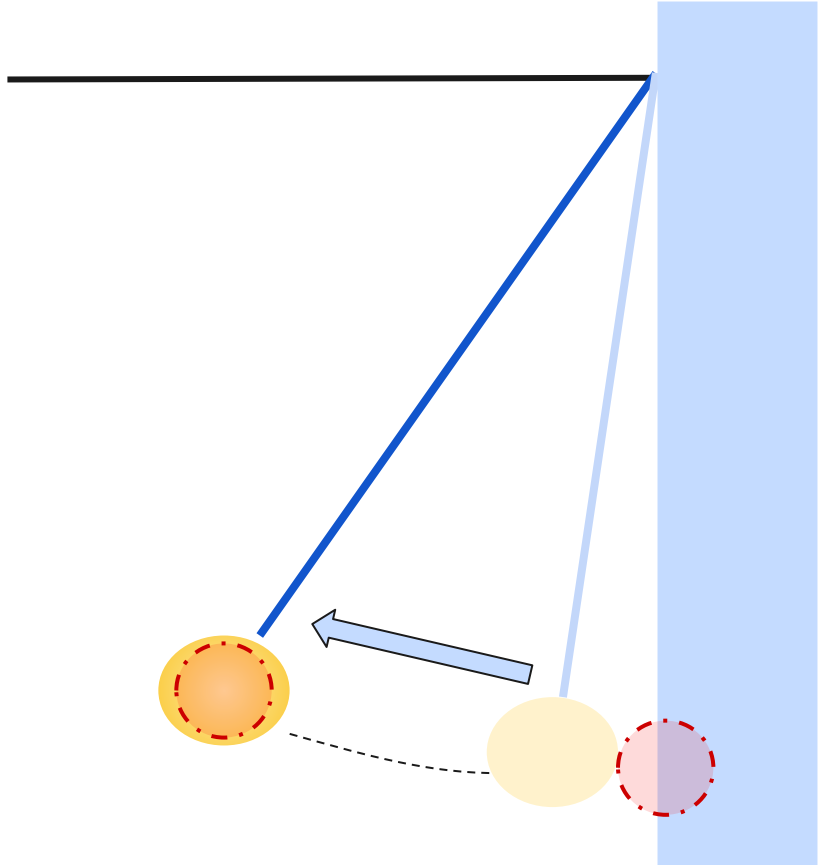
$$-\tau = I\alpha = I\frac{d\omega}{dt}$$

$$\text{for } t = 0,\ \omega = \omega_k$$

$$t = \frac{I}{\tau}(\omega_k - \omega)$$

$$\text{for } \omega = 0,\ t = t_{k1}$$

$$t_{k1} = \frac{I\omega_k}{\tau}$$



Part 2

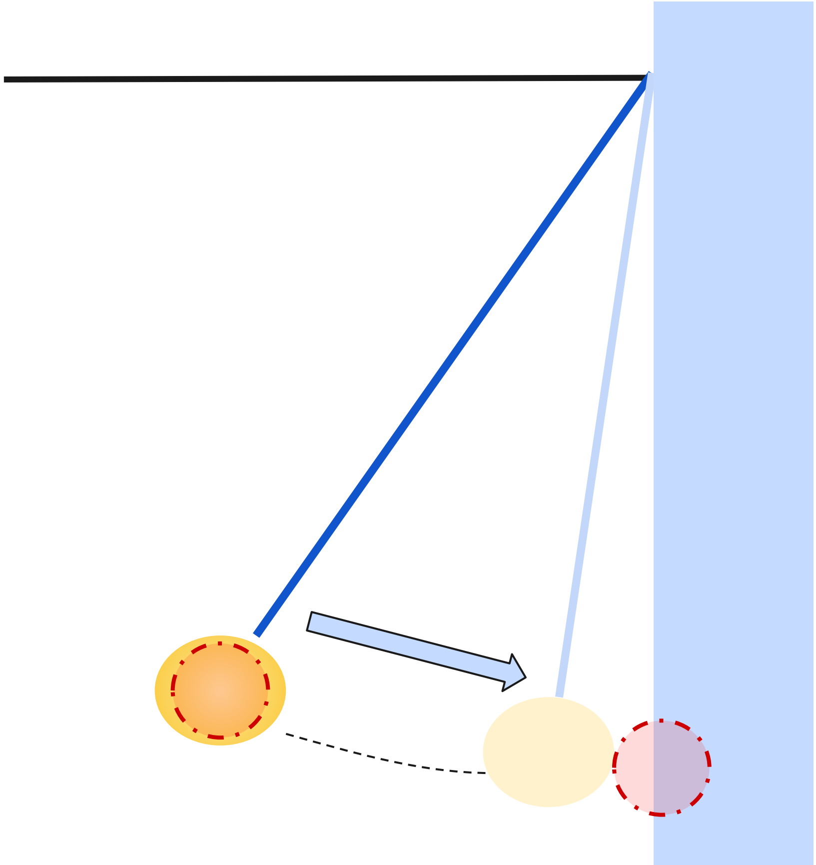
$$\tau = I\alpha = I\frac{d\omega}{dt}$$

For $t = 0$, $\omega = 0$

$$t = \frac{I}{\tau}\omega$$

For $\omega = \omega_k$, $t = t_{k2}$

$$t_{k2} = \frac{I\omega_k}{\tau}$$



Time between successive collisions is:

$$t_k = t_{k1} + t_{k2} = 2\frac{Iw_k}{\tau}$$

$$= \frac{2I}{\tau}\sqrt{\frac{2c \cdot KE_{k-1}}{I}}$$

where $KE_{k-1} = \frac{1}{2}I\omega_{k-1}^2$

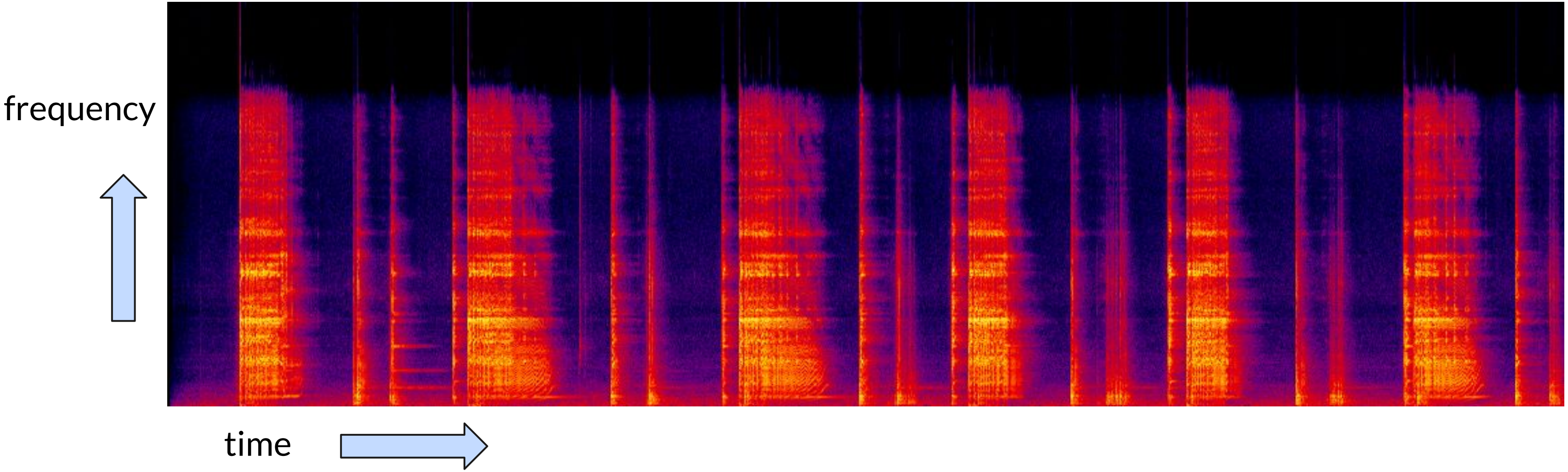
$$= \frac{2I}{\tau}\sqrt{cw_{k-1}^2}$$

$$= \sqrt{c}t_{k-1}$$

$$= (\sqrt{c})^{k-1}t_0$$

since $\frac{t_k}{t_{k-1}} = \sqrt{c}$

Set up and obtaining Spectrograms



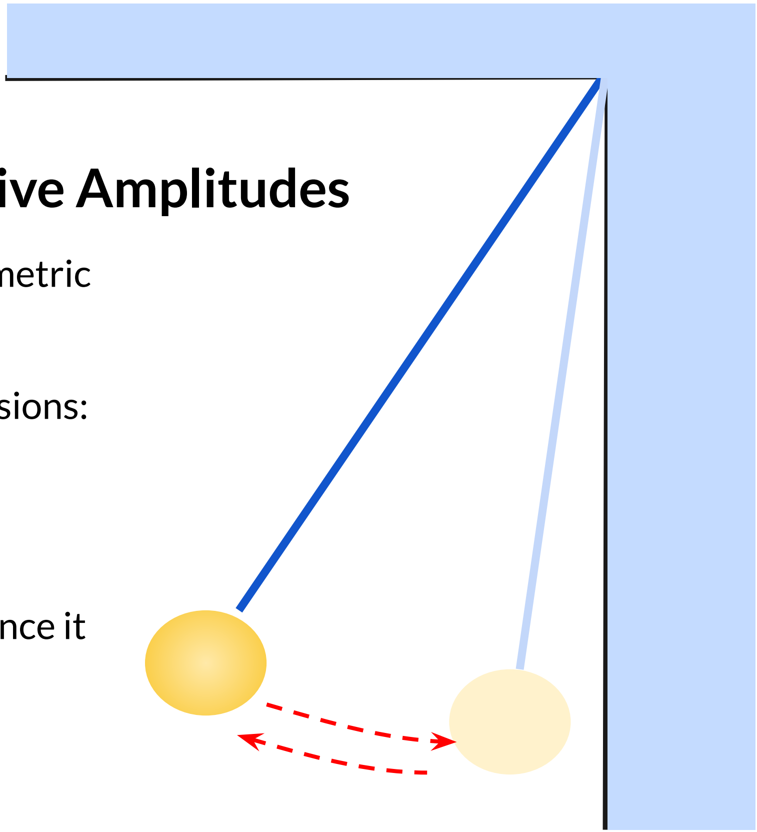
Expectations for times between Successive Amplitudes

- We preliminarily observed collision time periods to be geometric progressions, or constant consecutive ratios of terms.

- We obtain the expression for the time period between collisions:

$$t_k = (\sqrt{c})^{k-1} t_0$$

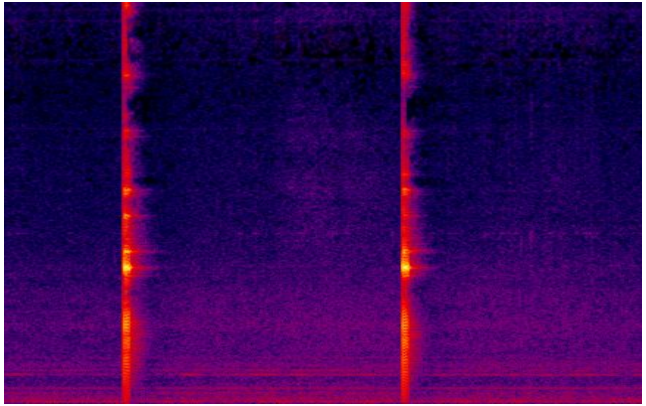
- We have investigated the time period over the amplitude since it is easier to quantify with our available resources.

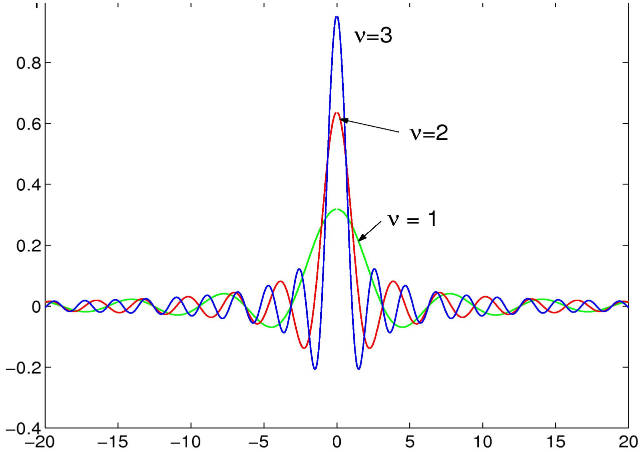


Nature of Collisions

We model each collision sound pulse at time **t** as a delta function of the form $\boldsymbol{\delta(x-x_t)}$

When decomposed into its fourier components, the delta function produces a large range of frequencies which we see in the figure.





Variation of fraction of Energy loss (e^2)

- e = coefficient of restitution
- S_y = dynamic yield strength (dynamic "elastic limit")
- E' = effective elastic modulus
- ρ = density
- v = velocity at impact
- μ = Poisson's ratio

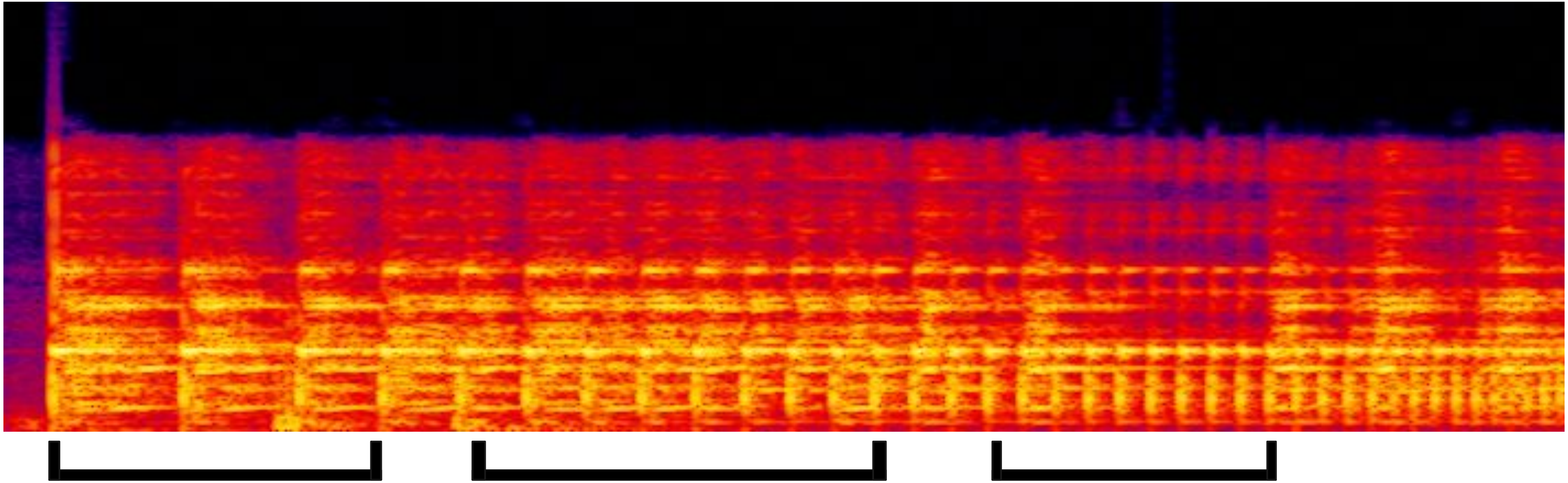
$$e = 3.1\left(\frac{S_y}{1}\right)^{\frac{5}{8}}\left(\frac{1}{E'}\right)^{\frac{1}{2}}\left(\frac{1}{v}\right)^{\frac{1}{4}}\left(\frac{1}{\rho}\right)^{\frac{1}{8}}$$

Notice that decrease in velocity leads to increase in **e** which is a decrease in the fraction of energy lost.

This leads to the following conclusion:

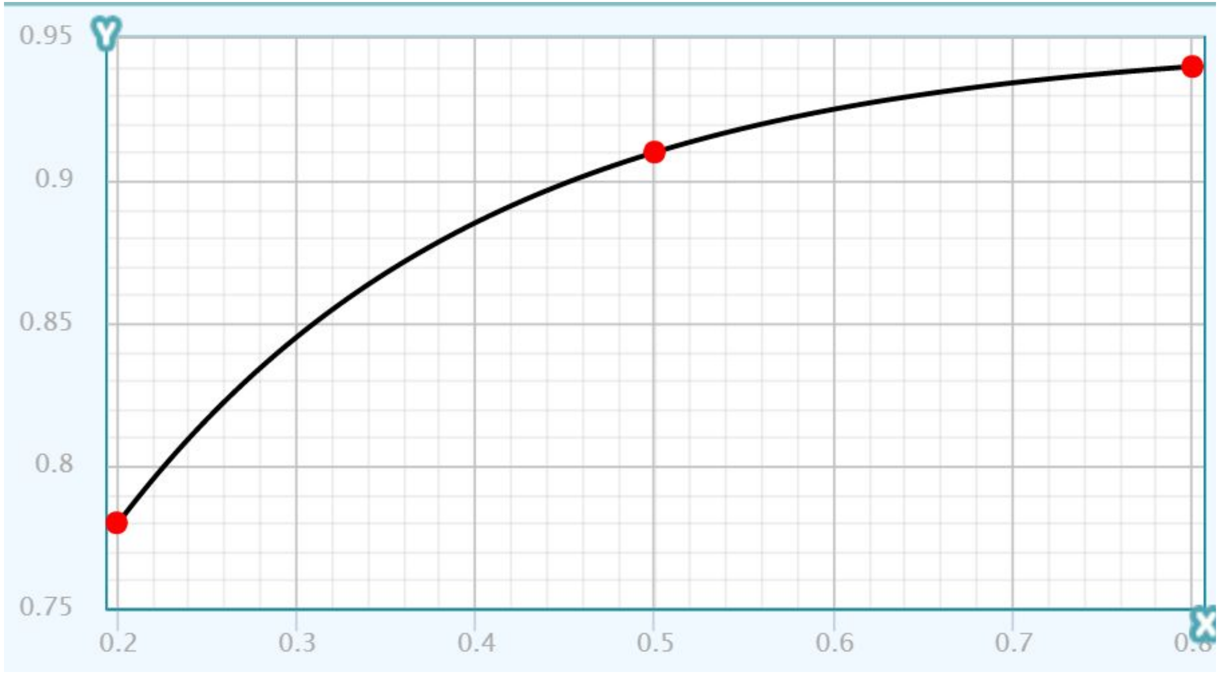
Locally, (i.e for small number of consecutive collisions), ***e*** *is constant and our simplified model works for the time analysis. But over time* ***e*** *changes enough to produce a noticeable change*

Display of local character in Data



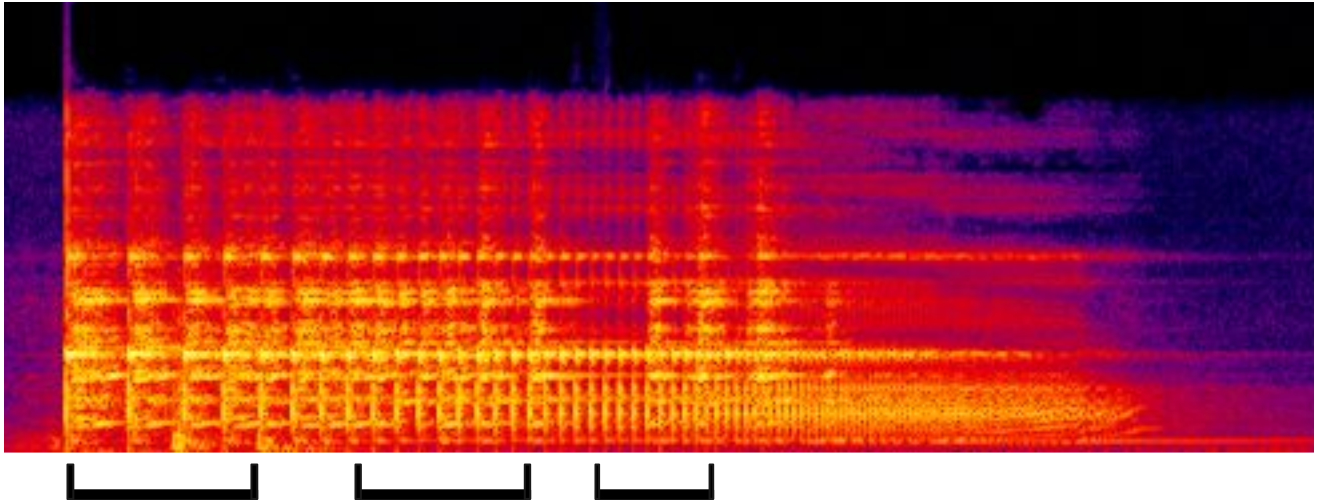
$C^{1/2} = 0.78$ $C^{1/2} = 0.91$ $C^{1/2} = 0.94$

$C^{1/2}$ (Fraction of Energy Loss)



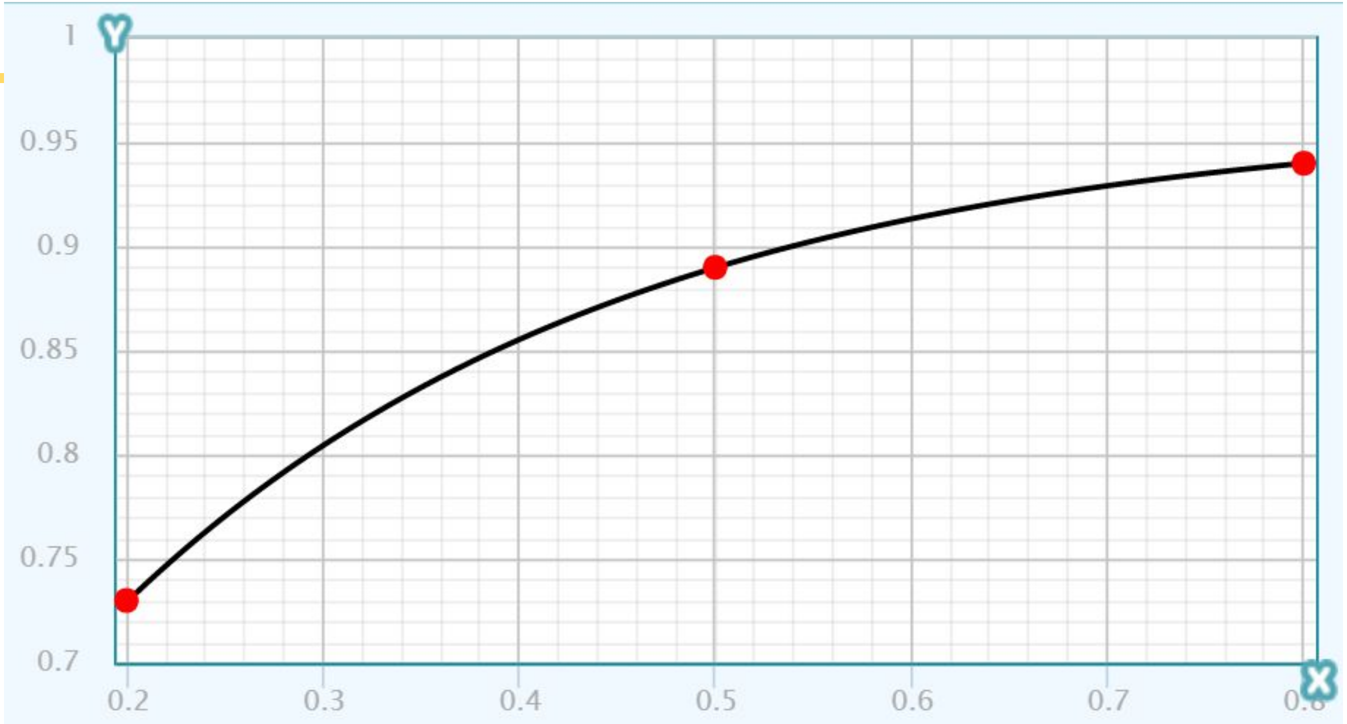
Time (not to scale)

Display of local character in Data



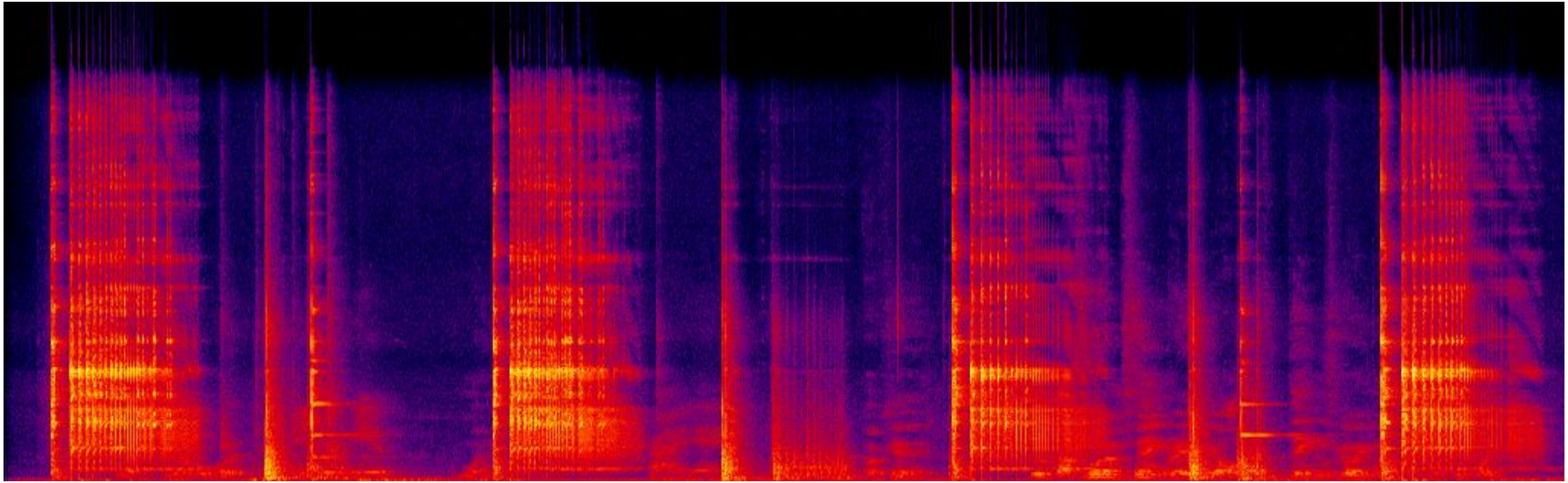
$C^{1/2} = 0.73$ $C^{1/2} = 0.89$ $C^{1/2} = 0.94$

$C^{1/2}$ (Fraction of Energy Loss)

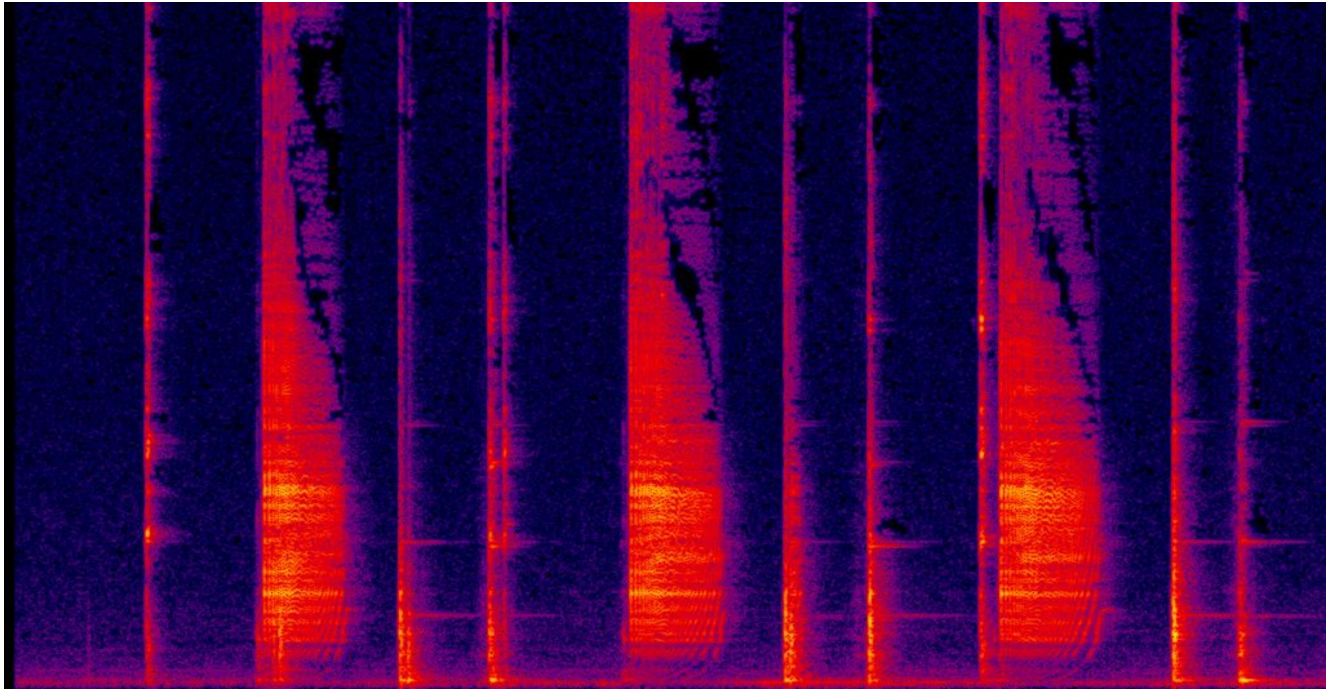


Time (not to scale)

Further Data with similar character



Original



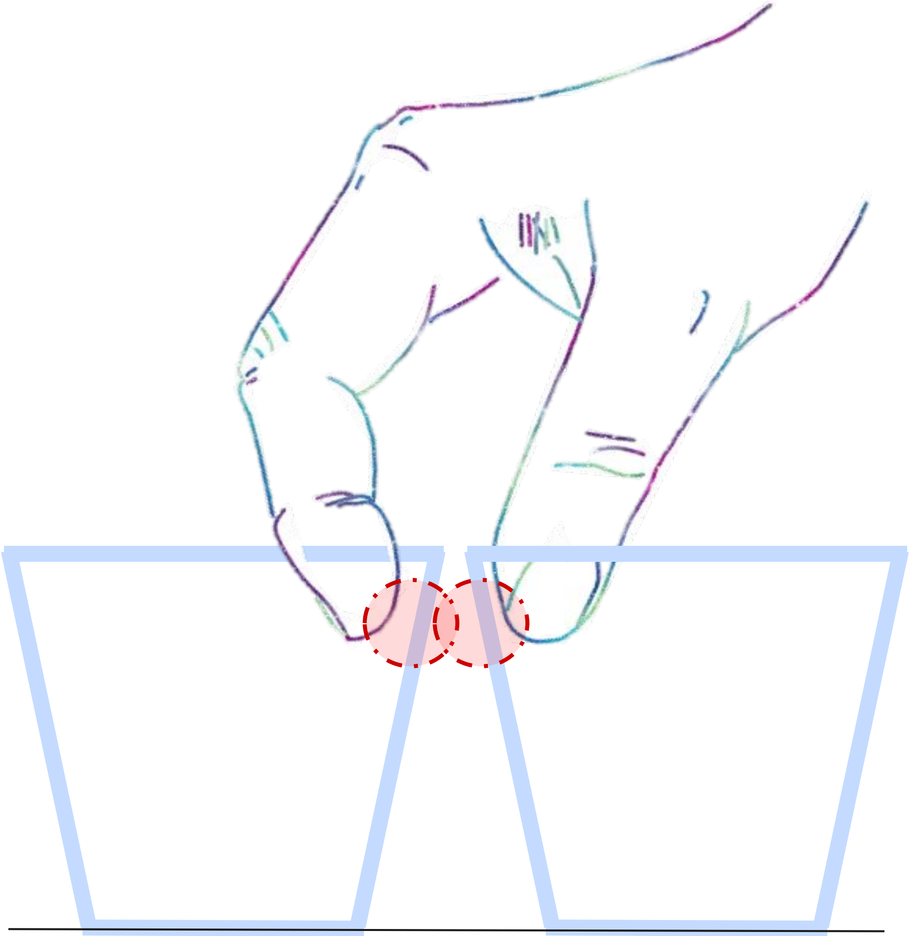
Calculation of Exact behaviour

By incorporating $e = 3.1\left(\frac{S_y}{1}\right)^{\frac{5}{8}}\left(\frac{1}{E'}\right)^{\frac{1}{2}}\left(\frac{1}{v}\right)^{\frac{1}{4}}\left(\frac{1}{\rho}\right)^{\frac{1}{8}}$

into our original equations instead of using constant **e** we can calculate the exact behaviour of the frequency. This can be easily modelled numerically.

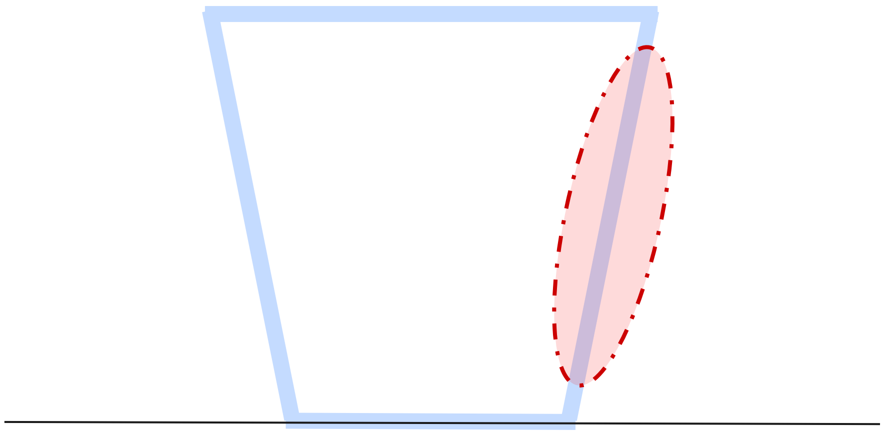
Summary of Assumptions

- Constant torque



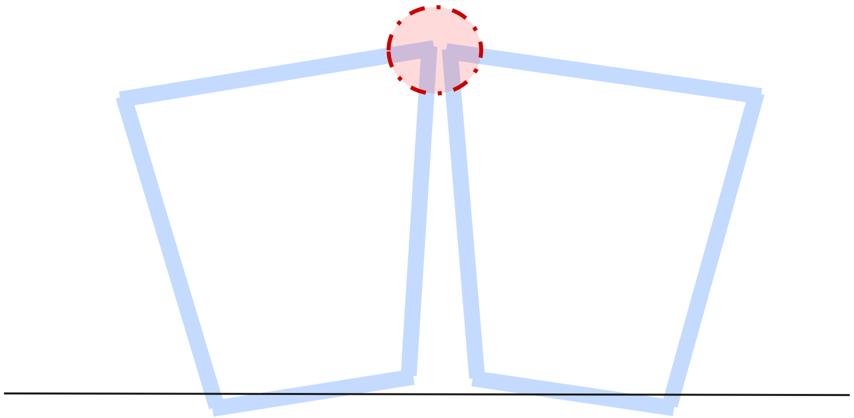
Summary of Assumptions

- Constant torque
- Rigid body (no change in shape)



Summary of Assumptions

- Constant torque
- Rigid body (no change in shape)
- Small angle approximation has been made freely in the analysis



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Further aspects of consideration

- Effect of temperature.
- Effect of modulus of material.
- Effect of liquid in glasses.

Thank you.

End trail (Hard to Reproduce)